Pipe networks - Handouts

Lecture 3. Pipe networks: pipe in series and in parallel

1 Pipe Networks

- In civil engineering applications pipes generally exist not singly but in systems. We now go on to study systems composed of networks of more than one pipe. This divides into systems of increasing complexity:
    - Pipes in series with other pipes of different properties such as diameter
    - Pipes in parallel with other pipes
    - Branched pipe systems
    - More complex networks
    - Pump-pipe systems
- The Powerpoint slides in class show examples of water distribution networks in a model and on a map, giving an idea of the complexity involved.
- Water distribution systems tend to have multiple loops so that supply can be maintained in the event of breakdown or maintenance.
- Sewer systems are more *dendritic*, branches converging on a main trunk sewer leading to a treatment plant or outfall.

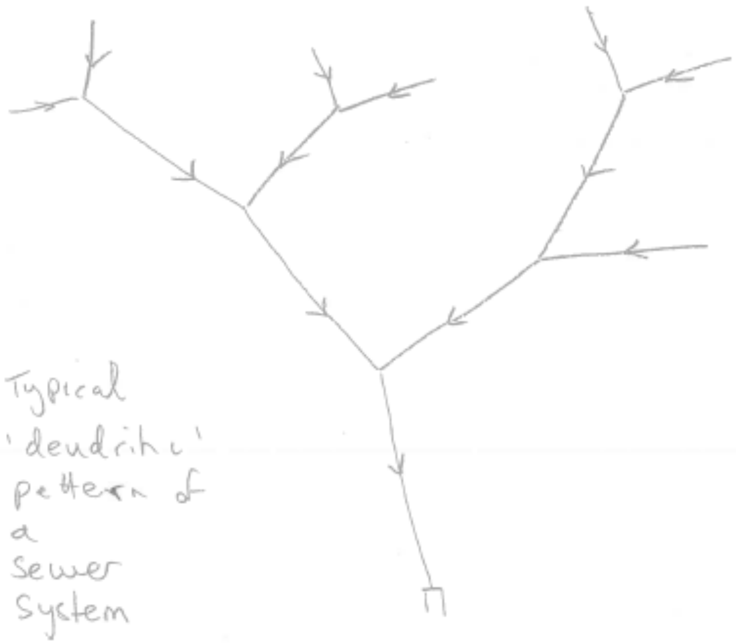


Figure 1

2 Pipes in Series

- The point about pipes in series is that the flow in each must be the same from continuity considerations (i.e. conservation of mass).
- The head loss in the system will be the sum of the head losses in the individual pipes within it.
- In the figure, note that the slope of the energy line is different for the two pipes: they are different diameters, so the velocity and hence the friction losses will also be different leading to a different friction slope.
- Q=Q1=Q2 and $H=h_{f1}+h_{f2}$

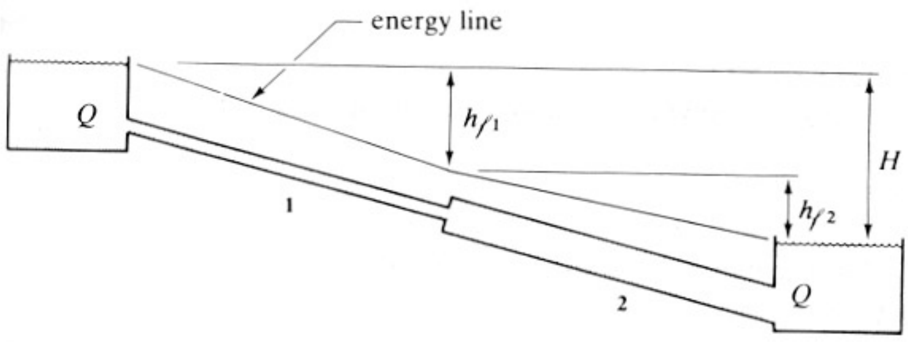


Figure 2

3 Pipes in Parallel

- For pipes in parallel, the total flow will be the sum of the flows in each pipe, but given that all pipes start and finish in the same place, the head loss will be the same in each.
- $Q=Q_1+Q_2$ and $H=h_{f1}=h_{f2}$

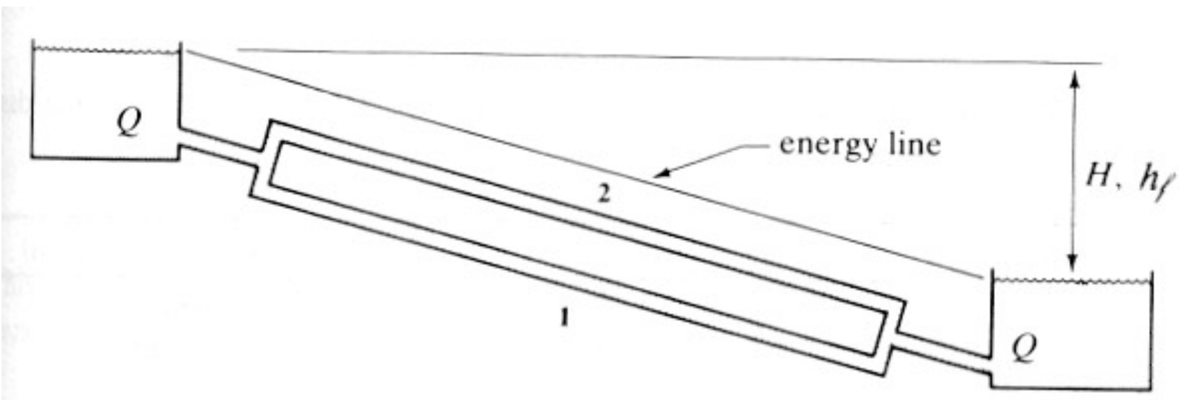


Figure 3

4. EXAMPLES

1. Supply and storage reservoirs

This is the classic and simplest case of verification problem for two reservoirs (Fig 4.1):

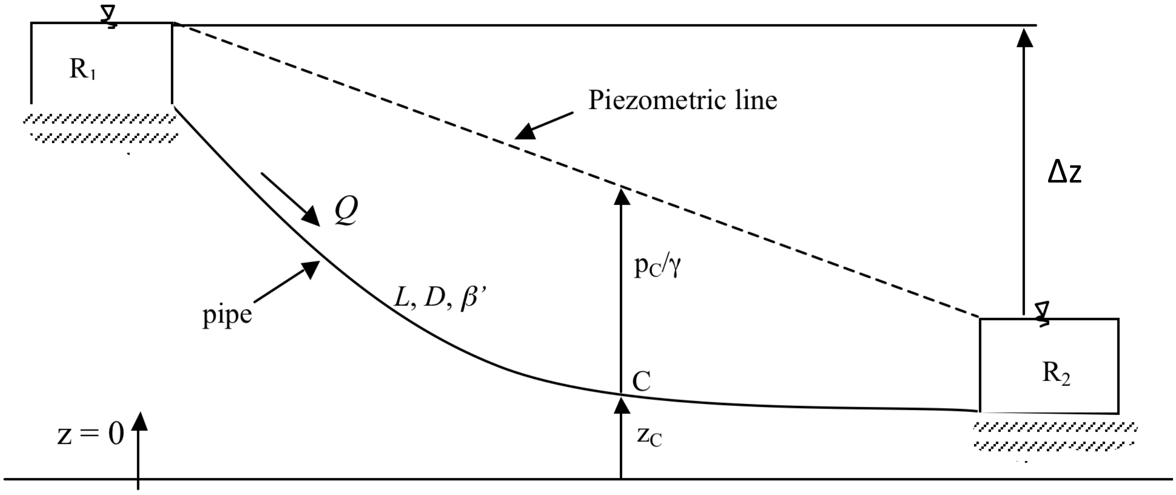


Figure 4.1

<u>Aim</u>: calculate the flow rate *Q* once the geometric characteristics such Δz, *L*, *D*, *β'* are known.

Using the Darcy's law, the equation of motion is

$$\Delta z = \beta' \frac{Q^2}{D^{5.33}} L$$

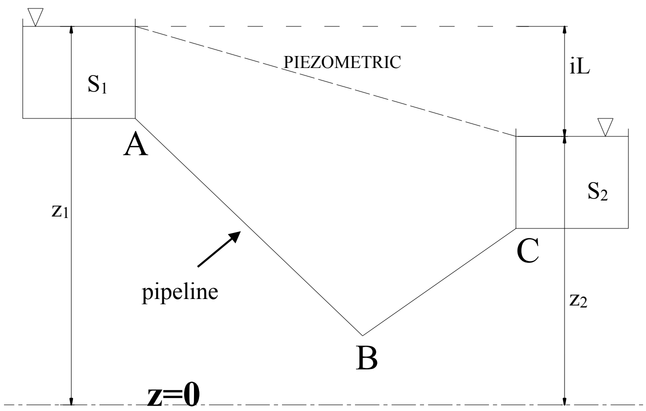
and then the flow rate is

$$Q = \sqrt{\frac{\Delta z D^{5.33}}{\beta' L}}$$

Notes: the pipe reach is subjected to a positive pressure in that the piezometric height in each section is given by the difference between the piezometric head *h* and the geodetic height *z* of the pipe centreline.

2. The 'used' pipes' criteria for pipelines network design

This criteria is simply based on the experience that the material of the pipe (especially in the case of metallic ducts) will become worn-out in time. This will change the pipe roughness and produces a consequent variation in the flow rate. For instance, used pipes tend to have a roughness that increases in time as more as they are used. Therefore this produces a decrease in the performance of the whole plant. To prevent future malfunctioning, the general criteria adopted in design operation is that of calculate the design diameter assuming for it a roughness equivalent to that expected if it was used.



First of all the equation regulating the motion of the flow rate through the pipeline has to be written in order to recognize the entity of the head losses per unit length *i*:

In order to calculate the best diameter *D* (from a pure hydraulics point of view), the used pipe criteria assures a good performance of the pipeline also in the future.

The head losses can be expressed by means of the Darcy formula,

$$i = \beta \frac{Q^2}{D^{5,33}},$$

The diameter D is therefore

$$i = \beta_{used} \cdot \frac{Q^2}{D^{5,33}} \longrightarrow D$$

FIRST CONSIDERATION

Such a value of diameter is not commercially available, thus a correct way of proceeding is to choose the two diameters D_1 and D_2 closest the calculated one,

The plant will be therefore built by using two reaches of different diameters. In doing this, the problem must be now re-organized to calculate the relative length pertaining to the different diameters

$$\begin{cases} L = L_1 + L_2 \\ z_1 - z_2 = i_1 L_1 + i_2 L_2 \end{cases},$$

where the head losses have to be written as

$$i_1 = \beta_{used} \cdot \frac{Q^2}{D_1^{5,33}}$$

$$i_2 = \beta_{used} \cdot \frac{Q^2}{D_2^{5,33}}$$

and inserted in the previous equations, to obtain the following solution:

$$L_1 \quad \leftrightarrow \quad D_1$$

$$L_2 \quad \leftrightarrow \quad D_2$$

NOTE: since we are dealing with pipe hydraulically long the local head loss due to the discontinuity in presence of the different diameters, is not considered.

SECOND CONSIDERATION

We must decide about the arrangement of the two reaches. Each solution has both advantages and disadvantages.

CASE 1. First L_1 D_1 and then L_2 D_2 (Figure 4.2)

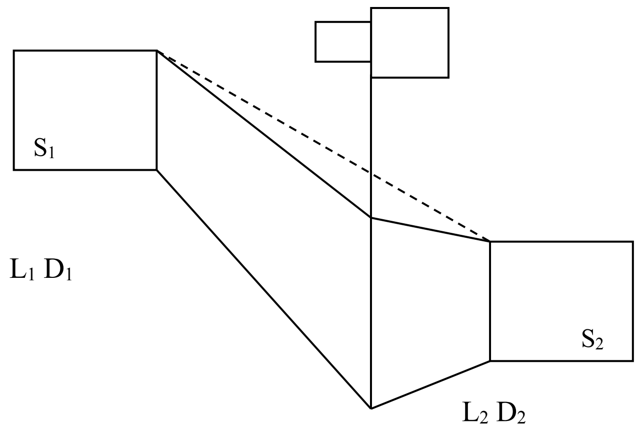


Figure 4.2

This choice is the most commonly used in that the piezometric head along the pipe immediately decreases. The advantage finds therefore a justification in the fact that the material is not too much stressed by high pressures, especially in proximity of critical points such junctions, etc.

CASE 2. First L_2 D_2 and then L_1 D_1 (Figure 4.3)

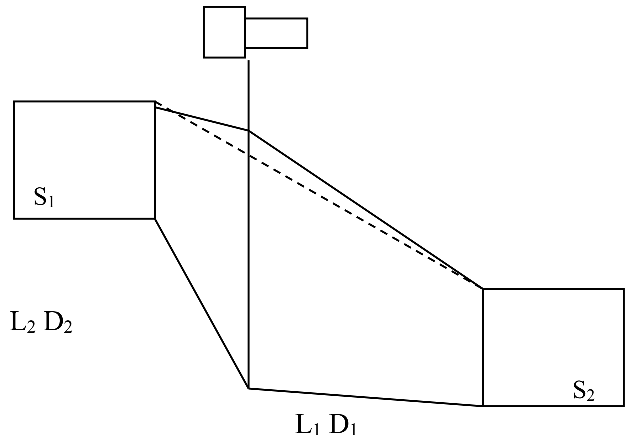


Figure 4.3

This arrangement is not very used, except in the case where morphological profiles would induce the piezometric to cross the pipeline, causing then a negative pressure over a given reach of pipe. Water distribution systems usually are not allowed to develop with reaches having a negative pressure in that locally infiltrations could pollute the transported water.

Let us choose then the first solution and go into some more considerations. The system is therefore the one of Figure 4.2

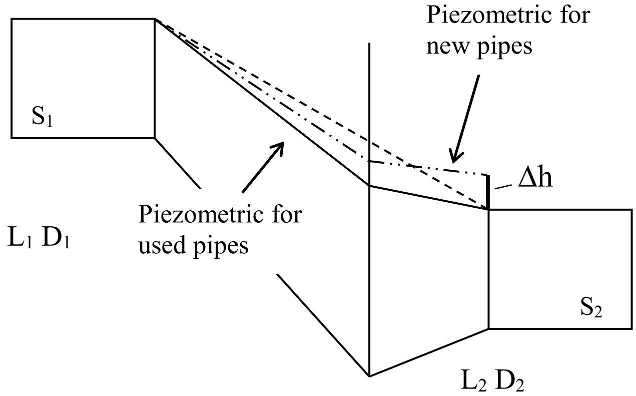


Figure 4.4

THIRD CONSIDERATION

Once the plant is built the pipes are, of course, new and therefore with the calculated diameters the conveyed flow rate will be generally higher than that effectively requested. Conversely, with new pipes, the requested flow rate Q, will pass with a piezometric less sloped than that of the used pipes (Figure 4.4).

A technical solution to this problem is that of introducing artificially a relevant localized head loss Δh in order to lower the piezometric up to the level of the Reservoir S2. The value of Δh can be calculated by an energy balance equation

$$\Delta z = \Delta h + iL = \Delta h + i_{1N} L_1 + i_{2N} L_2 \,,$$

where the distributed head losses are now calculated referring to the roughness of the new pipes,

$$i_{1N} = \beta_{new} \cdot \frac{Q^2}{D_1^{5,33}}$$

$$i_{2N} = \beta_{new} \cdot \frac{Q^2}{D_2^{5,33}}$$

thus obtaining the quantity that has to be dissipated, for instance by using a pressure regulating valve.

NOTE: of course this localized head loss is now relevant and has the scope of regulating the flow rate when the plant is new. As more as the roughness will increase due to the functioning, then the valve will be regulated (i.e. slowly opened) in order to reduce the local head loss to guarantee the same flow rate Q.

3. Distributed supply

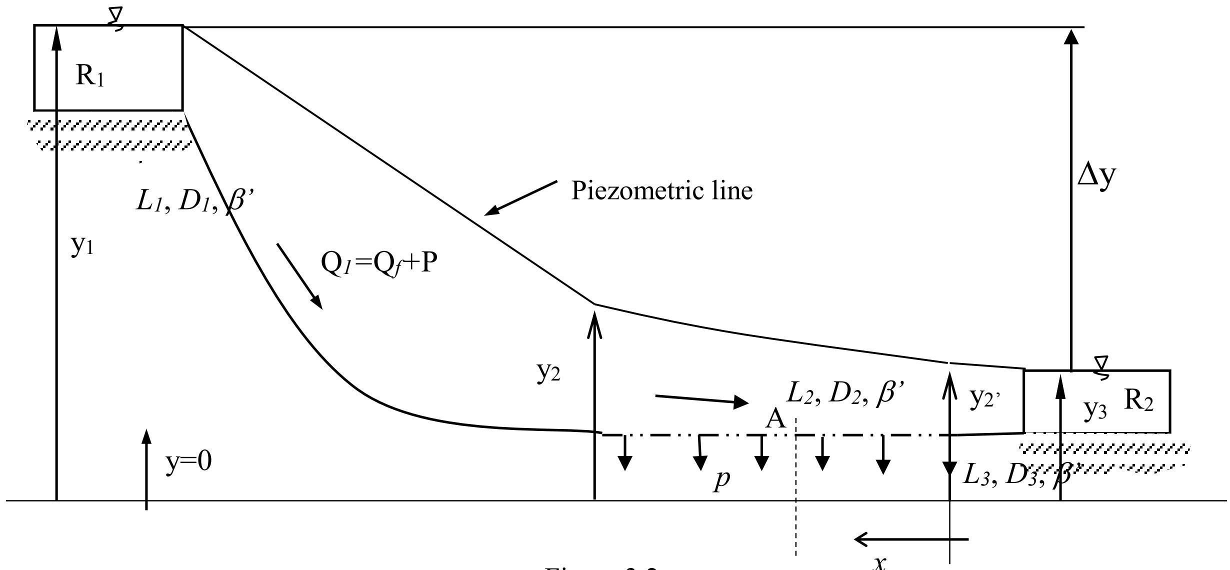


Figure 3.2

Along the reach L_2 there is a distributed user per unit length p equal to

$$p=P/L_2. \tag{3.1}$$

The aim is to study this system. Let us indicate with Q_f the flow rate coming to the reservoir R_2. The flow rate leaving the reservoir R_1 must therefore be equal to

$$Q_1=Q_f+P.$$

The flow rate in the generic section A can be written as

$$Q_a=Q_f+px.$$

For the first reach the equation of motion will be then

$$\Delta y_1 = y_1 - y_2 = \beta' \frac{(Q_f + P)^2}{D_1^{5.33}} L_1,$$

while for the third reach is

$$\Delta y_3 = y_{2'} - y_3 = \beta' \frac{(Q_f)^2}{D_3^{5.33}} L_3$$

Supposing that for each infinitesimal length *dx* of the pipe the Darcy formula can be assumed valid, then along the reach 2, the differential variation of the piezometric head can be written as

$$dh = \beta' \frac{(Q_f + px)^2}{D_2^{5.33}} dx \, .$$

The integral over the length L_2 furnish the analytical expression of the total variation for h, i.e.

$$\Delta h_2 = \int_0^{L_2} \beta' \frac{(Q_f + px)^2}{D_2^{5.33}} dx \, ,$$

which, developing the square term, becomes

$$\Delta h_2 = \int_0^{L_2} \beta' \frac{Q_f^{\,2} + p^2 x^2 + 2\, pxQ_f}{D_2^{5.33}} dx \, ,$$

which indicates clearly that the piezometric line along the reach 2-2’ is no longer rectilinear, but it is a curve. A first solution of the integral can be made supposing the diameter D_2 being constant for all the reach. In this case integrating and doing some algebra one obtains

$$\Delta h_2 = \frac{\beta'}{D_2^{5.33}} \left(Q_f^{\,2} L_2 + \frac{1}{3} p^2 L_2^{\,3} + 2\, pQ_f L_2^2 \right)$$

or, making use of the expression (3.1)

$$\Delta h_2 = \frac{\beta'}{D_2^{5.33}}\left(Q_f{}^2 L_2 + \frac{1}{3} P^2 L_2 + 2PQ_f L_2 \right).$$

The global equation of motion therefore is

$$\Delta h = y_1 - y_3 = \beta' \frac{(Q_f + P)^2}{D_1^{5.33}} L_1 + \frac{\beta'}{D_2^{5.33}}\left(Q_f{}^2 L_2 + \frac{1}{3} P^2 L_2 + 2PQ_f L_2 \right) + \beta' \frac{(Q_f)^2}{D_3^{5.33}} L_3 .$$

The interesting result appears when we would compare the two following situations:

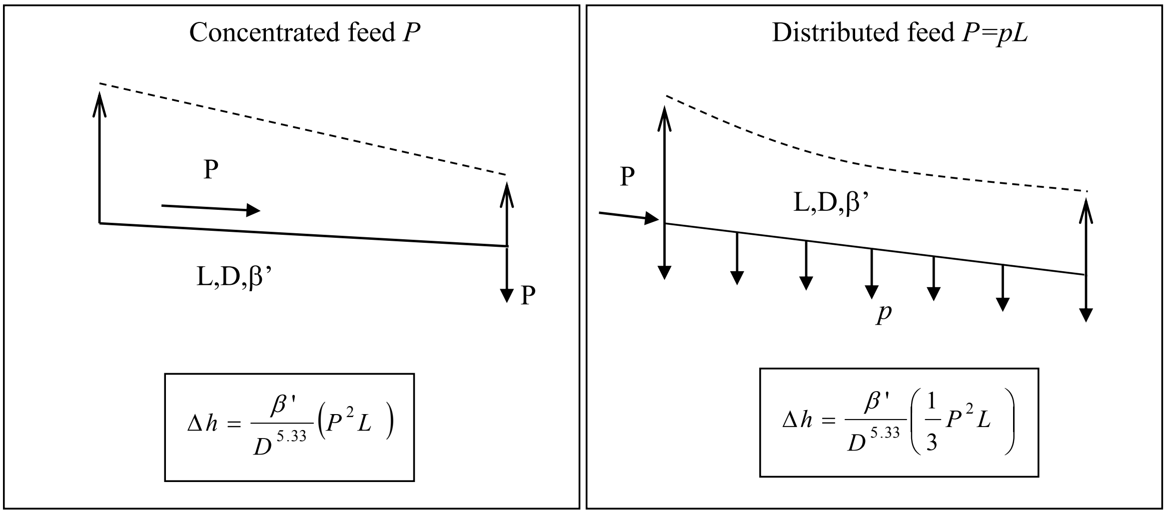


Figure 3.3

The head losses corresponding to a distributed user *P=pL* over a pipe of diameter *D* and length *L* are equal to 1/3 of the losses occurring in the case of concentrated user *P*.

4. Emergency supply to an external user, case 1

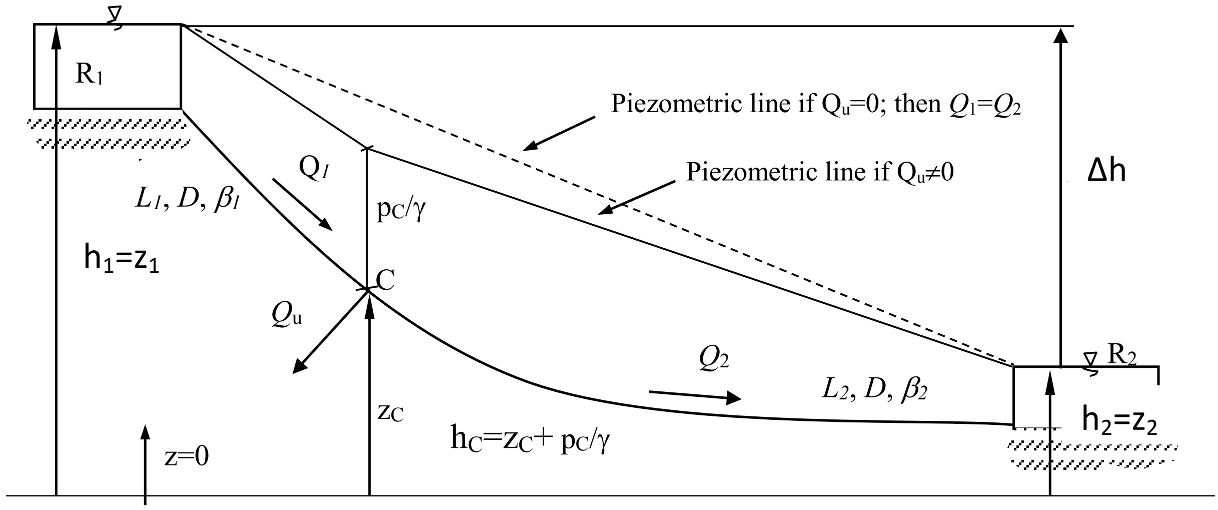


Figure 3.5

The suction of an arbitrary flow rate Q_u in a section C requires a decrease of the piezometric line in that section. Then we evaluate the new distribution of flow rates Q_1, Q_2. Equation of motion must be related to the motion within the reach 1 and 2. Furthermore, a continuity equation at the node C is useful to close the problem:

$$\begin{cases} h_1 - h_c = \beta_1 \dfrac{Q_1^2}{D^5} L_1 \\ h_c - h_2 = \beta_2 \dfrac{Q_2^2}{D^5} L_2 \, , \\ Q_2 = Q_1 - Q_u \end{cases}$$

which has the solution

$$Q_u = \sqrt{\frac{(h_1 - h_c)D^5}{\beta_1 L_1}} - \sqrt{\frac{(h_c - h_2)D^5}{\beta_2 L_2}} \, .$$

The maximum value for Q_u can be found imposing that p_C/γ is equal to zero in the node C, i.e.

$$Q_u^{MAX} = \sqrt{\frac{(h_1 - z_c)D^5}{\beta_1 L_1}} - \sqrt{\frac{(z_c - h_2)D^5}{\beta_2 L_2}}$$

5. Emergency supply to an external user, case 2

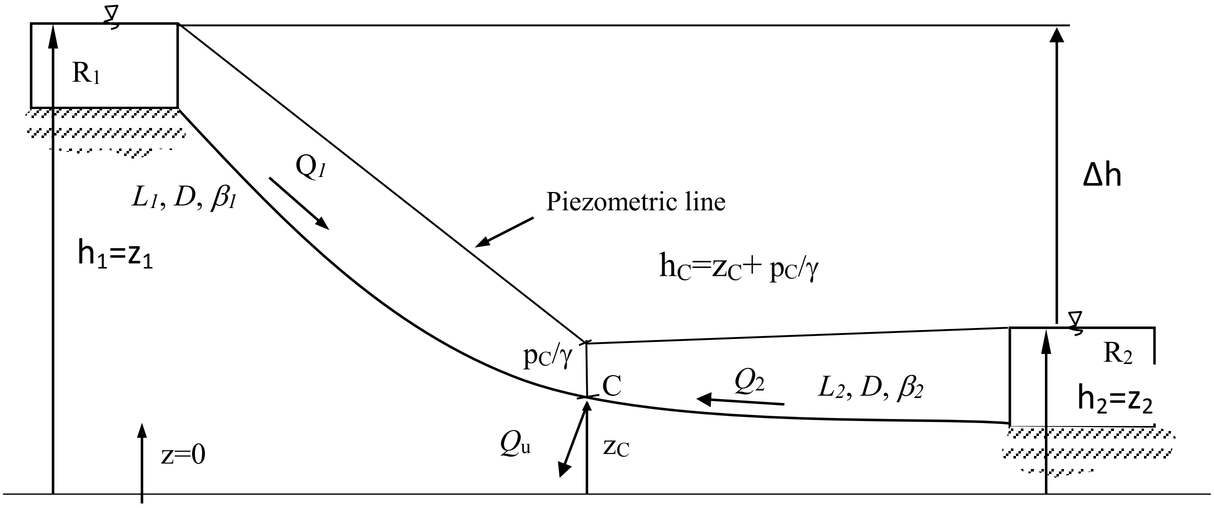


Figure 3.6

This problem is very similar to the previous one, but in this case both the two reservoirs contribute to supply the user. The continuity and motion equations are therefore

$$\begin{cases} h_1 - h_c = \beta_1 \dfrac{Q_1^2}{D^5} L_1 \\ h_2 - h_c = \beta_2 \dfrac{Q_2^2}{D^5} L_2 \,, \\ Q_u = Q_1 + Q_2 \end{cases}$$

which has the solution

$$Q_u = \sqrt{\frac{(h_1 - h_c) D^5}{\beta_1 L_1}} + \sqrt{\frac{(h_2 - h_c) D^5}{\beta_2 L_2}} \,.$$

In this case, the maximum flow rate (when $p_C/\gamma=0$) is higher than that of the previous case since both the two reservoirs contribute to the supply.

6. Emergency supply to a storage reservoir

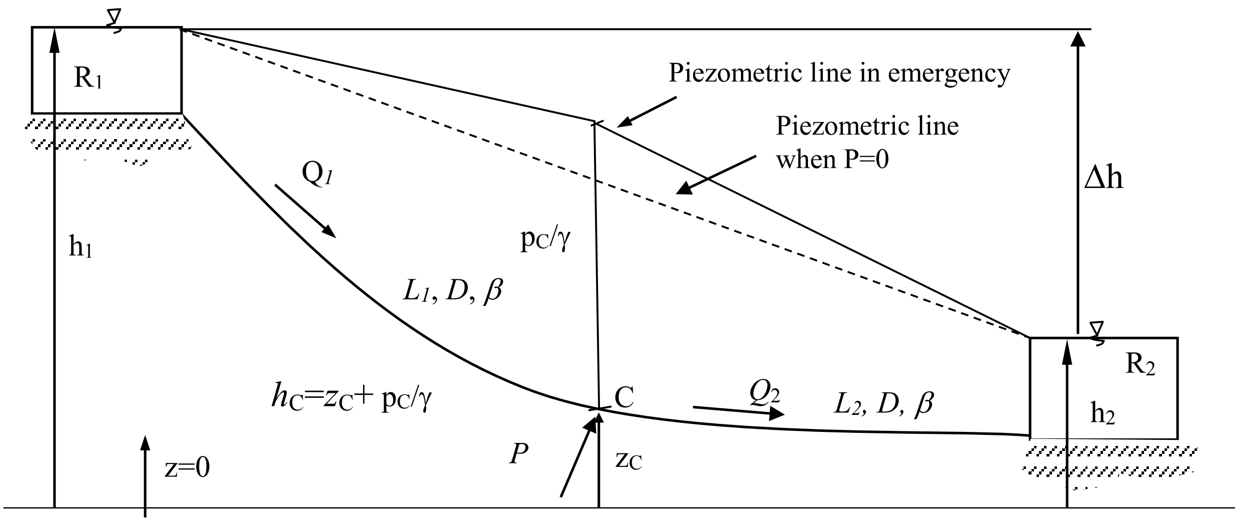


Figure 3.7

The normal functioning of the plant is for Q_1=Q_2 , where Q_2 is the flow rate requested by the storage reservoir R_2 . The piezometric line is a rectilinear line joining the two reservoirs (------).

In the case of emergency where the flow rate Q_2 is no longer guaranteed then is possible to introduce an extra flow rate *P* within the pipe (for example at the point indicated as C) in order to supply the requested flow rate Q_2.

The effect of an introduction of a given flow rate P, is that of producing across the section C an increase of the piezometric line, making the piezometric head h_c also unknown

We want to study, therefore, this scheme. For the first reach we have

$$\Delta h_1 = h_1 - h_C = \beta \frac{(Q_2 - P)^2}{D^n} L_1 ,$$

while for the second one going from the point C to the reservoir R2 the equation of motion is

$$\Delta h_2 = h_C - h_2 = \beta \frac{(Q_2)^2}{D^n} L_2 \, .$$

It has to be noted that the two equations above are sufficient to close the problem in that the global problem only has two unknowns, i.e. P and h_C .

The new flow rate Q_1 leaving the reservoir R1 will spontaneously adapt itself to the new configuration which account for the presence of the emergency supply P. Therefore this happen according to the continuity equation written at the node C:

$$Q_2 = Q_1 + P.$$